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THEORY OF PLASTICITY AND CREEP TAKING ACCOUNT OF MICROSTRAINS*

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A relationship between the theories of plasticity and creep of the type /1, 2/ and theories based on the concept of slip is set up. A most logical structure is proposed for the constitutive equations of the theory which is convenient for engineering calculations.

It has been shown /3/ that the theory of slip /4/ results from the theories /1, 2/. However, it remains unclear whether a deeper connection exists between these theories. Moreover, the connection between creep theories constructed using the approach in /1, 2/ and creep theories based on the slip concept was not generally examined. A survey of the development of polycrystalline strain theory /5/ yields a complete representation of the state of matters in plasticity and creep theories.

1. We consider the plasticity theory /1, 2/ proposed for investigating the plastic deformation of polycrystalline metals and based on the assumption that the statistics of anisotropic crystals can be replaced by the statistics of isotropic particles possessing different flow limits and a random field of initial microstresses and strains.

The theory of plasticity is constructed on the basis of the following assumption.

1°. A local plastic flow law is formulated that connects the stresses and strains; this law contains one or more random parameters.

2°. The joint distribution function of the random parameters is considered given and determined taking experimental data into account.

3°. The generalized Kröner relationships connecting the stress and stress deviations are assumed to be valid. Such relationships enable the local plastic strain laws to be connected with the macroscopic plastic strain laws.

Under active plastic strain the constitutive equations of the theory can be represented as follows /1, 2/

$$\langle\sigma_{ij}\rangle=\tau_{ij}+m\varepsilon_{ij}^p+\int_0^\infty\int_\Omega R(\tau,\tau',\lambda_{ij},\lambda_{ij}')\,\varepsilon_{ij}^p(\tau',\lambda_{ij}')\,d\Omega'\,d\Phi(\tau') \tag{1.1}$$

$$\tau_{ij}=\tau\mu_{ij},\quad \mu_{ij}=d\varepsilon_{ij}^p/d\lambda$$

$$\varepsilon_{ij0}^p=\varepsilon_p{}^\circ\lambda_{ij},\quad \lambda_{ij}\lambda_{ij}=1$$

$$d\lambda=\sqrt{d\varepsilon_{ij}^p\,d\varepsilon_{ij}^p},\quad \langle\varepsilon_{ij}^p\rangle=\int_0^\infty\int_\Omega\varepsilon_{ij}^p(\tau',\lambda_{ij}')\,d\Omega'\,d\Phi(\tau')$$

Here $\varepsilon_{ij0}{}^\circ$ is a random tensor of the initial microstrains, τ is the local yield point, λ_{ij} is a directional unit deviator fixing the direction in deviator space, Ω is a set of directions of the active microplastic strains, $d\Omega'$ is the differential form ("solid angle" in five-dimensional deviator space), $\Phi(\tau)$ is the integral distribution function of the local yield points, and $\langle\ \rangle$ is the averaging symbol.

Without considering the different versions of the theory (to which the surveys /6, 7/ are devoted) here, we note that they afforded the possibility of describing and even predicting fairly fine effects observed in tests.

2. Let us use the hypothesis /3/ $\mu=\lambda_{ij}$, $\varepsilon_{ij}^p=\varepsilon_p\lambda_{ij}$. In fact, it means the following. The local flow surfaces are planar, they move translationally under active loading; the plastic strains are directed along the normal to the plane flow surfaces. Then we have in conformity with (1.1)

$$\langle\sigma_{ij}\rangle=\tau_{ij}+m\varepsilon_{ij}^p(\tau,\lambda_{ij})+\int_0^\infty\int_\Omega R(\tau,\tau',\lambda_{ij},\lambda_{ij}')\times\varepsilon_{ij}^p(\tau',\lambda_{ij}')\,d\Omega'\,d\Phi(\tau') \tag{2.1}$$

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$$\tau_{ij} = \tau\lambda_{ij}, \quad \varepsilon_{ij}{}^p = \varepsilon_p \lambda_{ij} \tag{2.2}$$

$$\langle \varepsilon_{ij}{}^p \rangle = \int_0^\infty \int_\Omega \varepsilon_{ij}{}^p (\tau', \lambda_{ij}') \, d\Omega' \, d\Phi(\tau') \tag{2.3}$$

Eq.(2.1) holds only for those directions $\lambda_{ij} \in \Omega$ in which active microplastic strain $\varepsilon_p^{\cdot} > 0$ occurs. The flow condition can be obtained from (2.1) if it is multiplied by λ_{ij} and summation is performed taking conditions (2.2) into account. It has the form

$$\langle \sigma_{ij} \rangle \lambda_{ij} \leqslant \tau + m\varepsilon_p(\tau, \lambda_{ij}) + \int_0^\infty \int_\Omega R_1(\tau, \tau', \lambda_{ij}, \lambda_{ij}') \times \varepsilon_p(\tau', \lambda_{ij}') \, d\Omega' \, d\Phi(\tau') \tag{2.4}$$

$$(R_1 = R\lambda_{ij}\lambda_{ij}')$$

We note that the equality sign in (2.4) is achieved for the directions of the active microplastic strain $\lambda_{ij} \in \Omega$. It is convenient to introduce the new notation

$$\mathrm{T}(\tau, \lambda_{ij}) = \tau + m\varepsilon_p(\tau, \lambda_{ij}) + \int_0^\infty \int_\Omega R_1(\tau, \tau', \lambda_{ij}, \lambda_{ij}') \times \varepsilon_p(\tau', \lambda_{ij}') \, d\Omega' \, d\Phi(\tau') \tag{2.5}$$

We call the quantity $\mathrm{T}(\tau, \lambda_{ij})$ the intensity of the effective stresses, hence it is natural to call the function R_1 the influence function.

As is seen from (2.5), the intensity of the permissive stresses depends not only on the magnitude of the local plastic strains (the second component in (2.5)), but also on the microplastic strains of the remaining particles by means of the influence function. In the new notation, (2.4) takes the form

$$\langle \sigma_{ij} \rangle \lambda_{ij} \leqslant T(\tau, \lambda_{ij})$$

The differential flow condition can also be written

$$\langle \sigma_{ij}^{\cdot} \rangle \lambda_{ij} = \mathrm{T}^{\cdot}(\tau, \lambda_{ij})$$

in the active microplastic strain domain.

The last two conditions enable both the magnitude of the domain Ω that is under active loading, and the intensity of the microplastic strain rate $\varepsilon_T^{\cdot}$ to be determined.

We note that the intensity of the effective stresses can also be given in the following differential form

$$\mathrm{T}^{\cdot}(\tau, \lambda_{ij}) = m_1 \varepsilon_p^{\cdot}(\tau, \lambda_{ij}) + \int_0^\infty \int_\Omega R_2(\tau, \tau', \lambda_{ij}, \lambda_{ij}') \varepsilon_p^{\cdot}(\tau', \lambda_{ij}') \, d\Omega' \, d\Phi(\tau')$$

For a known magnitude of the microplastic strain rate intensity $\varepsilon_p^{\cdot}$ and a domain of active microplastic strain directions Ω the macroscopic strain is determined from the formula

$$\langle \varepsilon_{ij}^{p\cdot} \rangle = \int_0^\infty \int_\Omega \varepsilon_p^{\cdot}(\tau', \lambda_{ij}') \, d\Omega' \, d\Phi(\tau')$$

The case $\langle \tau \rangle = \tau = \tau_0$ also merits a separate examination. Then all the relationships presented are simplified and have the form

$$\langle \sigma_{ij} \rangle \lambda_{ij} \leqslant T(\lambda_{ij}), \quad \langle \sigma_{ij}^{\cdot} \rangle \lambda_{ij} = \mathrm{T}^{\cdot}(\lambda_{ij}) \tag{2.6}$$

Here

$$\mathrm{T}(\lambda_{ij}) = T_0 + m\varepsilon_p(\lambda_{ij}) + \int_\Omega R_1(\lambda_{ij}, \lambda_{ij}') \varepsilon_p(\lambda_{ij}') \, d\Omega' \tag{2.7}$$

or

$$\mathrm{T}^{\cdot}(\lambda_{ij}) = m_1 \varepsilon_p^{\cdot}(\lambda_{ij}) + \int_\Omega R_2(\lambda_{ij}, \lambda_{ij}') \varepsilon_p^{\cdot}(\lambda_{ij}') \, d\Omega' \tag{2.8}$$

$$\langle \varepsilon_{ij}{}^p \rangle = \int_\Omega \varepsilon_p(\lambda_{ij}') \lambda_{ij}' \, d\Omega', \quad \langle \varepsilon_{ij}{}^{p\cdot} \rangle = \int_\Omega \varepsilon_p^{\cdot}(\lambda_{ij}') \lambda_{ij}' \, d\Omega'$$

3. We will now show that under certain additional constraints, a number of known plasticity theories based on the slip concept is obtained from the theories in /1, 2/. We assume that the deviator has the particular representation

$$\lambda_{ij}=(n_i l_j+n_j l_i)/2 \tag{3.1}$$

where n_i and l_i, respectively, determine the normal to the area extracted in the material body and the direction to it. Then

$$\langle\sigma_{ij}\rangle\lambda_{ij}=\langle\sigma_{nl}\rangle,\quad \varepsilon_{ij}^{p}\lambda_{ij}=\varepsilon_p=\gamma_{nl}$$

Here $\langle\sigma_{nl}\rangle$ is the mean tangential stress to an area with normal n_i in the direction l_i. Taking account of the above, we write relationships (2.7) and (2.8) in the form

$$\langle\sigma_{nl}\rangle=T_0+m\gamma_{nl}+\int_\Omega R_1(n_i,n_i',l_i,l_i')\gamma_{nl}(n_i',l_i')\,d\Omega' \tag{3.2}$$

$$\langle\sigma_{nl}^{\cdot}\rangle=m_1\gamma_{nl}^{\cdot}+\int_\Omega R_2(n_i,n_i',l_i,l_i')\gamma_{nl}^{\cdot}(n_i',l_i')\,d\Omega' \tag{3.3}$$

We will examine certain special cases

$$1^\circ.\ m=0,\quad R_1=\delta(1-\lambda_{ij}\lambda_{ij}')(\langle\sigma_{nl}\rangle-T_0)/F(\langle\sigma_{nl}\rangle)$$

(F is the universal function of the material).

Then a classical version of the theory of slip /4/ is obtained from (3.2). Its disadvantages are obvious; there is no influence of shear in one direction on the change in shear in the other directions, and consequently, it is impossible to describe the Bauschinger effect and cyclic loading.

$$2^\circ.\ T_0=f(\sigma_i),\quad m=f(\sigma_i)r_1,\quad \sigma_i=\sqrt{\langle\sigma_{ij}\rangle\langle\sigma_{ij}\rangle}$$

$$R_1=f(\sigma_i)[r_2 l_i l_i'\delta(1-n_i n_i')+r_3\lambda_{ij}\lambda_{ij}']$$

In this case it follows from (3.2) that

$$\langle\sigma_{nl}\rangle=f(\sigma_i)\Big[1+r_1\gamma_{nl}+r_2\int_{\omega_1}^{\omega_2}\gamma_{nl}\cos(\omega_0-\omega)\,d\Omega'+r_3\langle\gamma_{nl}\rangle\Big]$$

$$\langle\gamma_{nl}\rangle=\langle\varepsilon_{ij}^{p}\rangle\lambda_{ij}$$

where ω_1,ω_2 are the boundaries of the slip fan in a plane with normal n_i, and r_1,r_2,r_3 are material constants. This version of the theory is proposed and developed in papers from the Leonov school /8, 9/. The theory mentioned already takes account of the relationship between the shears. However, the description of the cyclic loadings is fraught with serious difficulties and, as is noted in /9/, requires the introduction of a number od additional assumptions.

3°. We assume that $R_1(n_i,n_i',l_i,l_i')=R_3(n_i,n_i')$, $m\neq 0$. The maximum value of $\langle\sigma_{nl}\rangle$ for a fixed direction n_i is $\sigma_{ij}n_i s_i/s$, where s_i is the tangential stress vector acting on the area with normal n_i and s is the intensity of mean tangential stresses ($s=\sqrt{s_i s_i}$). Taking account of the above, we write (3.2) in the form

$$\langle s(n_i)\rangle=s+\int_\Omega R_3(n_i,n_i')\gamma_{nl}(n_i')\,d\Omega' \tag{3.4}$$

(Ω is the domain of active shear directions for different directions n_i). It is interesting that in such an approach the local shear orientation on the slip area coincides with the direction of the tangential stress acting on this area.

We now consider a special case of the representation (3.4). We set

$$R_3=\delta(1-n_i n_i')(\langle s(n_i)\rangle-s)/\gamma_p(\langle s\rangle)$$

then from (3.4) we arrive at Malmeister's theory /10, 11/.

If the relationships (3.3) are initial, then as in the preceding, we obtain

$$\langle s^{\cdot}(n_i)\rangle=\int_\Omega R_3(n_i,n_i')\gamma_{nl}^{\cdot}(n_i')\,d\Omega' \tag{3.5}$$

We assume that this last equation is solvable in the form

$$\gamma_{nl}^{\cdot}=\int_\Omega L(n_i,n_i')\langle s^{\cdot}(n_i)\rangle\,d\Omega' \tag{3.6}$$

Then a version of slip theory described in /12/ is obtained. The computational examples presented in /10-12/ showed that the theory can yield satisfactory agreement with test data.

Tests on cyclic varaible-sign loading are least favourable for the theory.

A function $R_3(n_i, n_i')$ can be constructed for the plane strain case so that the slip system would be planar /13/. In this case we arrive at the Leonov-Shvaiko theory /14/, which was later considerably developed in /13/.

Therefore, within the framework of the general approach proposed, the possibilities of different slip theory versions can be estimated and compared.

4. Making the theories (2.6)-(2.8) specific depends on specifying the influence function $R(\lambda_{ij}, \lambda_{ij}')$. It is natural to assume that the structure of this function has the simplest form

$$R(\lambda_{ij}, \lambda_{ij}') = R(\lambda_{ij}\lambda_{ij}') = a_1\delta(1-\lambda_{ij}\lambda_{ij}') + a_2\lambda_{ij}\lambda_{ij}' + a_3 + a_4\delta(1+\lambda_{ij}\lambda_{ij}') \tag{4.1}$$

where a_i are universal functions of the material dependent on the macromeasure of the plastic strain, and $\delta(1-\lambda_{ij}\lambda_{ij}')$ is the delta function.

We note that the case $a_3 = a_4 = 0$ is studied in /3/.

In the general case the constitutive equation of the theory takes the form (taking (4.1) into account)

$$\langle\sigma_{ij}^{\cdot}\rangle\lambda_{ij} = A_1\varepsilon_p^{\cdot}(\lambda_{ij}) + A_2\langle\varepsilon_{ij}^{p\cdot}\rangle\lambda_{ij} + A_3\int_\Omega \varepsilon_p^{\cdot}(\lambda_{ij}')\,d\Omega'$$

We introduce the deviator of the rate of change of the active stresses $\langle r_{ij}^{\cdot}\rangle = \langle\sigma_{ij}^{\cdot}\rangle - A_2\langle\varepsilon_{ij}^{\cdot}\rangle$. Then we obtain the following equation to find the rate of change in the microplastic strain intensity

$$A_1\varepsilon_p^{\cdot}(\lambda_{ij}) + A_3\int_\Omega \varepsilon_p^{\cdot}(\lambda_{ij}')\,d\Omega' = \lambda_{ij}\langle r_{ij}^{\cdot}\rangle$$

(Ω is the domain of active microplastic strain). The solution of this equation has the form

$$\varepsilon_p^{\cdot}(\lambda_{ij}) = \frac{1}{A_1}\lambda_{ij}\langle r_{ij}^{\cdot}\rangle - \frac{\mu}{A_1(1+\mu\Omega)}F_{ij}\langle r_{ij}^{\cdot}\rangle$$

$$F_{ij} = \int_\Omega \lambda_{ij}'\,d\Omega', \quad \mu = \frac{A_3}{A_1}$$

Hence

$$\langle\varepsilon_{ij}^{p}\rangle = \frac{1}{A_1}\left[G_{ijkl} - \frac{\mu}{1+\mu\Omega}F_{ij}F_{kl}\right]\langle r_{kl}^{\cdot}\rangle \tag{4.2}$$

$$G_{ijkl} = \int_\Omega \Lambda_{ij}'\lambda_{ke}'\,d\Omega'$$

Relationships (4.2) completely solve the problem of constructing the constitutive equations of the theory of microstrains for a known domain Ω of active microplastic strain directions. To determine the domain Ω itself it is necessary to use inequality (2.6). To investigate this inequality it is obviously necessary to construct an expression for the effective stress intensities $T(\lambda_{ij})$ and to analyse it in detail. The case of monotonic loading and the passage to a theory of flow with combined hardening is investigated most simply. Preliminary computations showed that the version of the theory presented describes complex loading fairly simply, including even cyclic loading. The change in the flow surface, especially its rear section, corresponds well with test data.

5. In recent years creep theories taking account of the microinhomogeneity of plastic strain development have been actively developed. Possible approaches to the construction of creep theories within the framework of the theories in /1, 2/ are presented in /6, 7/. Creep theories based on the slip concept have been studied in /9, 15-17/. Without examining the possible versions in creep theory construction, we present here a modified microstrain theory that follows from ideas in /1, 2, 7/ and we show the connection between such a version and creep theories using the concept of slip theory.

Following /7/, we take as the fundamental local law several modernized local plastic flow laws by considering that the process is spread out in time:

$$\tau_{ij} = \tau\mu_{ij}, \quad \mu_{ij} = d\dot{\varepsilon}_{ij}^{p}/d\lambda, \quad \tau \doteq \tau_0\psi(\lambda, \lambda^{\cdot}), \lambda^{\cdot} = \sqrt{\varepsilon_{ij}^{p\cdot}\varepsilon_{ij}^{p\cdot}} \tag{5.1}$$

$$\rho_{ij} = m\varepsilon_{ij}^{p} + \psi_a(\lambda, \lambda^{\cdot})\int_0^\infty\int_\Omega\int_0^t R(t-t', \tau_0, \tau_0', \lambda_{ij}, \lambda_{ij}')\times \varepsilon_{ij}^{p\cdot}(t', \tau_0', \lambda_{ij}')\,dt'\,d\Omega'\,d\Phi(\tau')$$

$$\tau_{ij} = \langle\sigma_{ij}\rangle - \rho_{ij}$$

We note that the law in the form (5.1) takes account of both the influence of the rate of plastic strain development on the local yield point of the material and on the hereditary properties of the material. It is mentioned in a number of papers /18, 19/ that versions of the theory when the local yield point τ and the influence function depend not only on the local but also on the macroscopic characteristics such as $\langle\lambda\rangle$, $\langle\lambda^{\cdot}\rangle$, Ω, merit special attention. These assumptions are not analysed in this paper.

If we go over to the simplest version of the theory when $\mu_{ij} = \lambda_{ij}$, $\varepsilon_{ij}^p = \varepsilon_p\lambda_{ij}$, we can obtain

$$\mathrm{T}(\tau_0, \lambda_{ij}, t) = \tau_0\psi(\varepsilon_p, \varepsilon_p^{\cdot}) + m\varepsilon_p(\tau_0, \lambda_{ij}, t) + \int_0^\infty\int_\Omega\int_0^t R(t-t', \tau_0, \tau_0', \lambda_{ij}, \lambda_{ij}')\,\varepsilon_p^{\cdot}\,dt'\,d\Omega'\,d\Phi(\tau')$$
$$\langle\sigma_{ij}\rangle\lambda_{ij} \leqslant \mathrm{T}(\tau_0, \lambda_{ij}, t)$$

For a known microplastic strain intensity function $\varepsilon_p(t, \tau_0, \lambda_{ij})$ the mean plastic strain is determined by means of (2.3).

The special case $\tau_0 = \langle\tau_0\rangle = T_0$ also merits attention. Then

$$T(t, \lambda_{ij}) \geqslant \langle\sigma_{ij}\rangle\lambda_{ij}$$
$$T(t, \lambda_{ij}) = T_0\psi(\varepsilon_p, \varepsilon_p^{\cdot}) + m\varepsilon_p(t, \lambda_{ij}) + \psi_1(\varepsilon_p, \varepsilon_p^{\cdot})\int_\Omega\int_0^t R(t-t', \lambda_{ij}, \lambda_{ij}')\,\varepsilon_{ij}^{p'}\,dt'\,d\Omega'$$

Let us trace the connection between slip type theories and theories (5.1). We again take the deviator λ_{ij} in the particular form (3.1), and then find from (5.1)

$$\langle\sigma_{nl}\rangle = T_{nl}(n_i, l_i, t) \tag{5.2}$$
$$T_{nl} = T_0\psi(\gamma_{nl}, \gamma_{nl}^{\cdot}) + m\gamma_{nl} + \psi_1(\gamma_{nl}, \gamma_{nl}^{\cdot}) \times \int_\Omega\int_0^t R(t-t', n_i, n_i', l_i, l_i')\,\gamma_{nl}'^{\cdot}\,dt'\,d\Omega'$$
$$\langle\varepsilon_{ij}^p\rangle = \int_\Omega \gamma_{nl}'\lambda_{ij}'\,d\Omega'$$

We will examine certain special cases 1°. $\psi = 1$, $\psi_1 = \psi_1(q)$ (q is the homological temperature)

$$\tau_0 = f(I, \langle\sigma_i\rangle), \quad I = \int_0^t G(t-t')\langle\sigma_{ij}^{\cdot}\rangle\,dt'$$
$$m = \psi_1 r_1, \quad R = R_1(n_i, n_i', l_i, l_i')$$

We then have from (5.2)

$$\langle\sigma_{nl}\rangle = f(I, \langle\sigma_i\rangle) + \gamma_{nl}\psi_1 r_1 + \psi_1\int_\Omega R_1(n_i, n_i', l_i, l_i')\,\gamma_{nl}(l_i', n_i')\,d\Omega'$$

Now, if we take

$$R_1 = r_2\delta(1 - n_i n_i')\,l_i l_i' + r_3\lambda_{ij}\lambda_{ij}'$$

in the last equation, we arrive at the creep theory proposed in /9/.

2°. $\psi = 1$, $T_0 = f(I, \langle\sigma_i\rangle)$, $\psi_1 = r$, $R = k(q, t-t')\,\delta(1 - n_i n_i')\cdot l_i l_i'$

Then

$$\langle\sigma_{nl}\rangle = f(I, \langle\sigma_i\rangle) + r\int_0^t\int_{\omega_1}^{\omega_2} k(q, t-t')\cos(\omega_0 - \omega)\,d\omega\,dt' \tag{5.3}$$

where ω_1, ω_2 are the boundaries of the set of slip directions in the plane with normal n_i. Eq.(5.3) determines the theory examined in /16/.

$$3^\circ.\ \psi = 1, \quad \tau_0 = q(1 - f(t)), \quad m = 0, \quad f(t) = \int_0^t Q(t-t')\,y(t')\,dt'$$
$$R = G(t-t')\,R_1(n_i, n_i', l_i, l_i')$$

(y is the rate of change of the temperature). We then obtain from (5.2)

$$\langle\sigma_{nl}\rangle = q(1 - f(t)) + \int_{\Omega} R_1(n_i, n_i', l_i, l_i') P_{nl}\, d\Omega'$$

$$P_{nl} = \int_0^t G(t - t')\gamma_{nl}^{\cdot}(t')\, dt'$$

We give the function $G(x)$ in the form $G(x) = \alpha e^{-\beta x}$. Then

$$P_{nl}^{\cdot} + \beta P_{nl} = \alpha\gamma_{nl}^{\cdot}$$

For such a representation of the function G we arrive at the creep theory proposed in /15/.

4°. We consider a version of theory (5.2) for

$$\psi = \psi_1 = (\gamma_{nl}^{\cdot}/a_{nl}^{\cdot})^m n(\gamma_{nl})$$

Then we find from (5.2)

$$\gamma_{nl}^{\cdot} = a_{nl}^{\cdot}(\langle\sigma_{nl}\rangle/g(\gamma_{nl}))^{1/m}$$

which corresponds to the theory proposed in /17/.

The versions of slip theory presented above are capable, in their authors' opinion, of a fairly good description of complex loading processes. The possibilities of the different creep theory versions must still be compared; however, it can already be said that the approach proposed by the authors of this paper possesses extensive possibilities and permits not only a comparison of existing versions of the theory but also a description of the phenomena occurring during creep, by simpler means. Certain results of this paper are briefly described in /20/.

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ON SYNTHESIS IN A DIFFERENTIAL GAME*

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The control problem is considered with minimization of the guaranteed result for a system described by an ordinary differential equation in the presence of uncontrolled noise. The concepts and formulation of the problem in /1/ are used. It is shown that, when forming the optimal control by the method of programmed stochastic synthesis /1-3/, the extremal shift at the accompanying point /1, 4/ can be reduced to extremal shift agianst the gradient of the appropriate function. This explains the connection between the programmed stochastic synthesis and the generalized Hamilton-Jacobi equation /5, 6/ in the theory of differential games.

1. Formulation of the problem. Consider the system described by the differential equation

$$x^{\cdot} = A(t)x + f(t, u, v),\ u \in P,\ v \in Q,\ t_0 \leqslant t \leqslant \vartheta \tag{1.1}$$

Here, x is the n-dimensional phase vector of the object, u is the r-dimensional control vector, v is the s-dimensional noise vector, $A(t)$ is a continuous matrix function, $f(t, u, v)$ is a continuous vector function, P and Q are compacta, and

$$\gamma = \int\limits_{[t_*, \vartheta]} \sigma(t, x[t])\mu(dt) + \int\limits_{t_*}^{\vartheta} \chi(t, u[t], v[t])\,dt \tag{1.2}$$

The functional which characteristizes the quality of the process in an interval $[t_*, \vartheta] \subset [t_0, \vartheta]$ is given. Here, $\sigma(t, x)$ and $\chi(t, u, v)$ are scalar continuous functions, $\sigma(t, x)$ satisfies a Lipschitz condition and is convex with respect to x, and $\mu(T)$ is the Borel measure in sets $T \subset [t_0, \vartheta]$.

We consider motions $x[t_*[\cdot]\vartheta] = \{x[t], t_* \leqslant t \leqslant \vartheta\}$, lying in a given bounded domain G of space $\{t, x\}$. Domain G is defined for $t_0 \leqslant t \leqslant \vartheta$, is closed, and satisfies the following condition /1, pp.37-42/. Given any initial position $\{t_*, x_*\} \in G$, every possible motion $x[t_*[\cdot]\vartheta]$ satisfies the inclusion $\{t, x[t]\} \in G$ for all $t \in [t_*, \vartheta]$. The problem is to construct the optimal strategy $u^\circ(\cdot) = \{u^\circ(t, x, \varepsilon), \{t, x\} \in G, \varepsilon > 0\}$, which gives the minimum guaranteed result $\rho^\circ(t_*, x_*)$.

This strategy exists and by definition, satisfies the following condition /1, pp.67-81/. Given any number $\zeta > 0$, a number $\varepsilon(\zeta) > 0$ and a function $\delta(\zeta, \varepsilon) > 0$ exist such that the control law

$$U = \{u^\circ(\cdot), \varepsilon, \Delta\{t_i\}\} \tag{1.3}$$

which forms the motion as a solution of the step-by-step differential equation

$$\begin{gathered} x^{\cdot}[t] = A(t)x[t] + f(t, u^\circ(t_i, x[t_i], \varepsilon), v[t]) \\ t_i \leqslant t < t_{i+1},\ i = 1, \ldots, k,\ t_1 = t_*,\ t_{k+1} = \vartheta,\ x[t_*] = x_* \end{gathered} \tag{1.4}$$

guarantees the inequality $\gamma \leqslant \rho^\circ(t_*, x_*) + \zeta$, no matter what the measurable noise

*Prikl.Matem.Mekhan.,50,6,898-902,1986